

Sfermion Mass Relations in Orbifold Family Unification

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(Received September 11, 2007)

We derive relations among sfermion masses based on orbifold family unification models. Sfermion mass relations are specific to each model and can be useful for a selection of realistic model.

§1. Introduction

Supersymmetric grand unified theories (SUSY GUTs) on an orbifold have attractive features as a realistic model beyond the minimal supersymmetric standard model (MSSM). The triplet-doublet splitting of Higgs multiplets is elegantly realized in the framework of SUSY $SU(5)$ GUT in five dimensions.¹⁾²⁾ Four-dimensional chiral fermions are generated through the dimensional reduction. Those phenomena originates from the fact that a part of zero modes are projected out by orbifolding, i.e., by non-trivial boundary conditions (BCs) concerning extra dimensions on bulk fields. There is a possibility that a (complete) family unification is realized by eliminating all mirror particles from the low-energy spectrum. Mirror particles are particles with opposite quantum numbers under the standard model (SM) gauge group G_{SM} .

Recently, the family unification has been studied in SUSY $SU(N)$ GUTs defined on a five-dimensional space-time $M^4 \times (S^1/Z_2)$.^{3)***)}

Here M^4 is the four-dimensional Minkowski space-time and S^1/Z_2 is the one-dimensional orbifold. A great variety of models have been found, in which zero modes from a single bulk field and a few brane fields compose three families, and we refer them as orbifold family unification models. At present, it is important to make powerful predictions in order to specify models by using experimental data. Much works concerning mass relations among scalar particles have been carried out based on the motivation that relations specific to each model will give a hint to understand the structure of the MSSM and beyond in four-dimensional SUSY models.^{7)–15)†)} Sum rules among sfermion masses have been also derived in two kinds of orbifold family unification models, and it has been pointed out that they can be useful probes

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^{***)} The possibility that one might achieve the complete family unification utilizing an orbifold has been also suggested in the earlier reference⁴⁾ in a different context. In Ref. 5), three families have been derived from a combination of a bulk gauge multiplet and a few brane fields. In Ref. 6), they have been realized as composite fields.

^{†)} Scalar mass relations have been examined in four-dimensional superstring models.^{16), 17)}

of each model.^{18)*)}

In this paper, we study sfermion masses from a general framework, based on orbifold family unification models under some assumptions regarding the breakdown of SUSY and gauge symmetries, and derive relations among them.

This paper is organized as following. In §2, we explain the outline of orbifold family unification models. In §3, we give a generic mass formula for sfermions and derive specific relations among sfermion masses. §4 is devoted to conclusions and discussions.

§2. Orbifold family unification

First we review the arguments in Ref.3). We study $SU(N)$ gauge theory on $M^4 \times (S^1/Z_2)$ with the gauge symmetry breaking pattern, $SU(N) \rightarrow SU(3) \times SU(2) \times SU(r) \times SU(s) \times U(1)^n$, which is realized by the Z_2 parity assignment

$$P_0 = \text{diag}(+1, +1, +1, +1, +1, -1, \dots, -1, -1, \dots, -1), \quad (2.1)$$

$$P_1 = \text{diag}(+1, +1, +1, -1, -1, \underbrace{+1, \dots, +1}_r, \underbrace{-1, \dots, -1}_s), \quad (2.2)$$

where $s = N - 5 - r$ and $N \geq 6$. The n is an integer which depends on the breaking pattern. The matrices P_0 and P_1 stand for the representation matrices (up to sign factors) of the fundamental representation for the Z_2 transformation ($y \rightarrow -y$) and the Z'_2 transformation ($y \rightarrow 2\pi R - y$), respectively. Here y is a coordinate of S^1/Z_2 and R is a radius of S^1 . After the breakdown of $SU(N)$, the rank k totally antisymmetric tensor representation $[N, k]$, whose dimension is ${}_N C_k$, is decomposed into a sum of multiplets of the subgroup $SU(3) \times SU(2) \times SU(r) \times SU(s)$

$$[N, k] = \sum_{l_1=0}^k \sum_{l_2=0}^{k-l_1} \sum_{l_3=0}^{k-l_1-l_2} ({}_3 C_{l_1}, {}_2 C_{l_2}, {}_r C_{l_3}, {}_s C_{l_4}), \quad (2.3)$$

where l_1, l_2 and l_3 are intergers, $l_4 = k - l_1 - l_2 - l_3$ and our notation is that ${}_n C_l = 0$ for $l > n$ and $l < 0$. Here and hereafter we use ${}_n C_l$ instead of $[n, l]$ in many cases. We list $U(1)$ charges for representations of subgroups in Table I. The $U(1)$ charges are those in the following subgroups,

$$SU(5) \supset SU(3) \times SU(2) \times U(1)_1, \quad (2.4)$$

$$SU(N-5) \supset SU(r) \times SU(N-5-r) \times U(1)_2, \quad SU(N-5-1) \times U(1)_2, \quad (2.5)$$

$$SU(N) \supset SU(5) \times SU(N-5) \times U(1)_3, \quad (2.6)$$

up to normalization. We assume that $G_{SM} = SU(3) \times SU(2) \times U(1)_1$ up to normalization of the hypercharge. Particle species are identified with the SM fermions by the gauge quantum numbers. The $(d_R)^c, l_L, (u_R)^c, (e_R)^c$ and q_L stand for down-type anti-quark singlets, lepton doublets, up-type anti-quark singlets, positron-type

*) Sfermion masses have been studied from the viewpoint of flavor symmetry and its violation in $SU(5)$ SUSY orbifold GUT.¹⁹⁾

Table I. The $U(1)$ charges for representations of fermions.

species	representation	$U(1)_1$	$U(1)_2$	$U(1)_3$
$(\nu_R)^c, \hat{\nu}_R$	$(3C_0, 2C_0, rC_{l_3}, sC_{k-l_3})$	0	$(N-5)l_3 - rk$	$-5k$
$(d'_R)^c, d_R$	$(3C_1, 2C_0, rC_{l_3}, sC_{k-l_3-1})$	-2	$(N-5)l_3 - r(k-1)$	$N-5k$
$l'_L, (l_L)^c$	$(3C_0, 2C_1, rC_{l_3}, sC_{k-l_3-1})$	3	$(N-5)l_3 - r(k-1)$	$N-5k$
$(u_R)^c, u'_R$	$(3C_2, 2C_0, rC_{l_3}, sC_{k-l_3-2})$	-4	$(N-5)l_3 - r(k-2)$	$2N-5k$
$(e_R)^c, e'_R$	$(3C_0, 2C_2, rC_{l_3}, sC_{k-l_3-2})$	6	$(N-5)l_3 - r(k-2)$	$2N-5k$
$q_L, (q'_L)^c$	$(3C_1, 2C_1, rC_{l_3}, sC_{k-l_3-2})$	1	$(N-5)l_3 - r(k-2)$	$2N-5k$
$(e'_R)^c, e_R$	$(3C_3, 2C_0, rC_{l_3}, sC_{k-l_3-3})$	-6	$(N-5)l_3 - r(k-3)$	$3N-5k$
$(u'_R)^c, u_R$	$(3C_1, 2C_2, rC_{l_3}, sC_{k-l_3-3})$	4	$(N-5)l_3 - r(k-3)$	$3N-5k$
$q'_L, (q_L)^c$	$(3C_2, 2C_1, rC_{l_3}, sC_{k-l_3-3})$	-1	$(N-5)l_3 - r(k-3)$	$3N-5k$
$l_L, (l'_L)^c$	$(3C_3, 2C_1, rC_{l_3}, sC_{k-l_3-4})$	-3	$(N-5)l_3 - r(k-4)$	$4N-5k$
$(d_R)^c, d'_R$	$(3C_2, 2C_2, rC_{l_3}, sC_{k-l_3-4})$	2	$(N-5)l_3 - r(k-4)$	$4N-5k$
$(\hat{\nu}_R)^c, \nu_R$	$(3C_3, 2C_2, rC_{l_3}, sC_{k-l_3-5})$	0	$(N-5)l_3 - r(k-5)$	$5N-5k$

lepton singlets and quark doublets. The particles with prime are regarded as mirror particles and expected to have no zero modes. Each fermion has a definite chirality, e.g. $(d_R)^c$ is left-handed and d_R is right-handed. Here the subscript L (R) represents the left-handedness (right-handedness) for Weyl fermions. The $(d_R)^c$ represents the charge conjugate of d_R and transforms as left-handed Weyl fermions under the four-dimensional Lorentz transformation.

A fermion with spin 1/2 in five dimensions is regarded as a Dirac fermion or a pair of Weyl fermions with opposite chiralities in four dimensions. The left-handed Weyl fermion and the corresponding right-handed one should have opposite Z_2 parity to each other, from the requirement that the kinetic term is invariant under the Z_2 parity transformation. We define the Z_2 parity of the representation $(_pC_{l_1}, _qC_{l_2}, _rC_{l_3}, _sC_{l_4})_L$ as follows,

$$\mathcal{P}_0 = (-1)^{l_1+l_2}(-1)^k\eta_k, \quad \mathcal{P}_1 = (-1)^{l_1+l_3}(-1)^k\eta'_k, \quad (2.7)$$

where η_k and η'_k are the intrinsic Z_2 parity. By definition, η_k and η'_k take a value +1 or -1. We list the Z_2 parity assignment for species in Table II. Note that mirror particles have the Z_2 parity $\mathcal{P}_0 = -(-1)^k\eta_k$. Hence *all zero modes of mirror particles can be eliminated by a choice of Z_2 parity when we take $(-1)^k\eta_k = +1$* . Hereafter we consider such a case.

We denote the flavor numbers of $(d_R)^c, l_L, (u_R)^c, (e_R)^c, q_L$ and (heavy) neutrino singlets as $n_{\bar{d}}, n_l, n_{\bar{u}}, n_{\bar{e}}, n_q$ and $n_{\bar{\nu}}$. Not only left-handed Weyl fermions but also right-handed ones, having even Z_2 parities $\mathcal{P}_0 = \mathcal{P}_1 = +1$, compose chiral fermions in the SM. When we take $(-1)^k\eta'_k = +1$, the flavor number of each chiral fermions are given by

$$n_{\bar{d}} = \sum_{i=1,4} \sum_{l_3=0,2,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}, \quad (2.8)$$

$$n_l = \sum_{i=1,4} \sum_{l_3=1,3,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}, \quad (2.9)$$

Table II. The Z_2 parity assignment for representations of fermions.

species	representation	\mathcal{P}_0	\mathcal{P}_1
$(\nu_R)^c$	$({}_3C_0, 2C_0, rC_{l_3}, sC_{k-l_3})_L$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$\hat{\nu}_R$	$({}_3C_0, 2C_0, rC_{l_3}, sC_{k-l_3})_R$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(d'_R)^c$	$({}_3C_1, 2C_0, rC_{l_3}, sC_{k-l_3-1})_L$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
l'_L	$({}_3C_0, 2C_1, rC_{l_3}, sC_{k-l_3-1})_L$	$-(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
d_R	$({}_3C_1, 2C_0, rC_{l_3}, sC_{k-l_3-1})_R$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$(l_L)^c$	$({}_3C_0, 2C_1, rC_{l_3}, sC_{k-l_3-1})_R$	$(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(u_R)^c$	$({}_3C_2, 2C_0, rC_{l_3}, sC_{k-l_3-2})_L$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$(e_R)^c$	$({}_3C_0, 2C_2, rC_{l_3}, sC_{k-l_3-2})_L$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
q_L	$({}_3C_1, 2C_1, rC_{l_3}, sC_{k-l_3-2})_L$	$(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
u'_R	$({}_3C_2, 2C_0, rC_{l_3}, sC_{k-l_3-2})_R$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
e'_R	$({}_3C_0, 2C_2, rC_{l_3}, sC_{k-l_3-2})_R$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(q'_L)^c$	$({}_3C_1, 2C_1, rC_{l_3}, sC_{k-l_3-2})_R$	$-(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$(e'_R)^c$	$({}_3C_3, 2C_0, rC_{l_3}, sC_{k-l_3-3})_L$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(u'_R)^c$	$({}_3C_1, 2C_2, rC_{l_3}, sC_{k-l_3-3})_L$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
q'_L	$({}_3C_2, 2C_1, rC_{l_3}, sC_{k-l_3-3})_L$	$-(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
e_R	$({}_3C_3, 2C_0, rC_{l_3}, sC_{k-l_3-3})_R$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
u_R	$({}_3C_1, 2C_2, rC_{l_3}, sC_{k-l_3-3})_R$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$(q_L)^c$	$({}_3C_2, 2C_1, rC_{l_3}, sC_{k-l_3-3})_R$	$(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
l_L	$({}_3C_3, 2C_1, rC_{l_3}, sC_{k-l_3-4})_L$	$(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(d_R)^c$	$({}_3C_2, 2C_2, rC_{l_3}, sC_{k-l_3-4})_L$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
$(l'_L)^c$	$({}_3C_3, 2C_1, rC_{l_3}, sC_{k-l_3-4})_R$	$-(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$
d'_R	$({}_3C_2, 2C_2, rC_{l_3}, sC_{k-l_3-4})_R$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
$(\hat{\nu}_R)^c$	$({}_3C_3, 2C_2, rC_{l_3}, sC_{k-l_3-5})_L$	$-(-1)^k \eta_k$	$-(-1)^{l_3} (-1)^k \eta'_k$
ν_R	$({}_3C_3, 2C_2, rC_{l_3}, sC_{k-l_3-5})_R$	$(-1)^k \eta_k$	$(-1)^{l_3} (-1)^k \eta'_k$

$$n_{\bar{u}} = n_{\bar{e}} = \sum_{i=2,3} \sum_{l_3=0,2,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}, \quad (2.10)$$

$$n_q = \sum_{i=2,3} \sum_{l_3=1,3,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}, \quad (2.11)$$

$$n_{\bar{\nu}} = \sum_{i=0,5} \sum_{l_3=0,2,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}, \quad (2.12)$$

using the equivalence on charge conjugation. When we take $(-1)^k \eta'_k = -1$, we obtain formulae in which n_l is exchanged by $n_{\bar{l}}$ and n_q by $n_{\bar{u}} (= n_{\bar{e}})$ in Eqs. (2.8) - (2.11). The total number of (heavy) neutrino singlets is given by $n_{\bar{\nu},k}^{(+)} = \sum_{i=0,5} \sum_{l_3=1,3,\dots} rC_{l_3} \cdot N-5-rC_{k-i-l_3}$ for $(-1)^k \eta'_k = -1$.

For arbitrary $N(\geq 6)$ and r , the flavor numbers from $[N, k]$ with $((-1)^k \eta_k, (-1)^k \eta'_k) = (a, b)$ equal to those from $[N, N-k]$ with $((-1)^{N-k} \eta_{N-k}, (-1)^{N-k} \eta'_{N-k}) = (a, -b)$ if r is odd and the flavor numbers from $[N, k]$ with $((-1)^k \eta_k, (-1)^k \eta'_k) = (a, b)$ equal to those from $[N, N-k]$ with $((-1)^{N-k} \eta_{N-k}, (-1)^{N-k} \eta'_{N-k}) = (a, b)$ if r is even. We list the flavor number of each chiral fermion derived from $[N, k]$ ($N = 5, \dots, 9$ and $k = 1, \dots, [N/2]$) in Table III. Here $[*]$ stands for Gauss's symbol, i.e., $[N/2] = N/2$ if N is even and $[N/2] = (N-1)/2$ if N is odd. In the 8-th column, the numbers in the parenthesis are the flavor numbers of neutrino

singlets for $(-1)^k \eta'_k = -1$.

Table III. The flavor number of each chiral fermion with $(-1)^k \eta_k = (-1)^k \eta'_k = +1$.

representation	(p, q, r, s)	$n_{\bar{d}}$	n_l	$n_{\bar{u}}$	$n_{\bar{e}}$	n_q	$n_{\bar{\nu}}$ ($n_{\bar{\nu}}$ with $(-1)^k \eta'_k = -1$)
$[N, 1]$	$(3, 2, r, s)$	1	0	0	0	0	$s \ (r)$
$[N, 2]$	$(3, 2, r, s)$	s	r	1	1	0	${}_r C_2 + {}_s C_2 \ (rs)$
$[6, 3]$	$(3, 2, 1, 0)$	0	0	1	1	1	0 (0)
	$(3, 2, 0, 1)$	0	0	2	2	0	0 (0)
$[7, 3]$	$(3, 2, 2, 0)$	1	0	1	1	2	0 (0)
	$(3, 2, 1, 1)$	0	1	2	2	1	0 (0)
	$(3, 2, 0, 2)$	1	0	3	3	0	0 (0)
$[8, 3]$	$(3, 2, 3, 0)$	3	0	1	1	3	0 (1)
	$(3, 2, 2, 1)$	1	2	2	2	2	1 (0)
	$(3, 2, 1, 2)$	1	2	3	3	1	0 (1)
	$(3, 2, 0, 3)$	3	0	4	4	0	1 (0)
$[8, 4]$	$(3, 2, 3, 0)$	1	1	3	3	3	0 (0)
	$(3, 2, 2, 1)$	2	0	2	2	4	0 (0)
	$(3, 2, 1, 2)$	1	1	3	3	3	0 (0)
	$(3, 2, 0, 3)$	2	0	6	6	0	0 (0)
$[9, 3]$	$(3, 2, 4, 0)$	6	0	1	1	4	0 (4)
	$(3, 2, 3, 1)$	3	3	2	2	3	3 (1)
	$(3, 2, 2, 2)$	2	4	3	3	2	2 (2)
	$(3, 2, 1, 3)$	3	3	4	4	1	1 (3)
	$(3, 2, 0, 4)$	6	0	5	5	0	4 (0)
$[9, 4]$	$(3, 2, 4, 0)$	1	4	6	6	4	1 (0)
	$(3, 2, 3, 1)$	4	1	4	4	6	0 (1)
	$(3, 2, 2, 2)$	3	2	4	4	6	1 (0)
	$(3, 2, 1, 3)$	2	3	6	6	4	0 (1)
	$(3, 2, 0, 4)$	5	0	10	10	0	1 (0)

Our four-dimensional world is assumed to be a boundary at one of the fixed points, on the basis of the ‘brane world scenario’. There exist two kinds of four-dimensional field in our low-energy theory. One is the brane field which lives only at the boundary and the other is the zero mode stemming from the bulk field. The Kaluza-Klein (KK) modes do not appear in our low-energy world because they have heavy masses of $O(1/R)$, the magnitude same as the unification scale M_U . There are many possibilities to derive three families from zero modes of (a few of) bulk field and suitable brane fields in the view point of chiral anomaly cancellation. Chiral anomalies may arise at the boundaries with the advent of chiral fermions. Those anomalies must be cancelled in the four-dimensional effective theory by the contribution of brane chiral fermions and/or counter terms such as the Chern-Simons term.^{20)–22)}

§3. Sfermion mass relations

We consider the SUSY version of $SU(N)$ models. In SUSY models, the hypermultiplet in the five-dimensional bulk is equivalent to a pair of chiral multiplets with opposite gauge quantum numbers in four dimensions. The chiral multiplet with the

representation $[N, N - k]$, which is a conjugate of $[N, k]$, contains a left-handed Weyl fermion with $[N, N - k]_L$. This Weyl fermion is regarded as a right-handed one with $[N, k]_R$ by using the charge conjugation. Hence our analysis in the previous section works on SUSY models.

We take the following assumptions in our analysis.

1. Three families in the MSSM come from zero modes of the bulk field with the representation $[N, k]$ and some brane fields.
2. We do not specify the mechanism that the $N = 1$ SUSY is broken down in four dimensions.^{*)} Soft SUSY breaking terms respect the gauge invariance.
3. Extra gauge symmetries are broken by Higgs mechanism at the same time as the orbifold breaking at the scale $M_U = O(1/R)$. Then the D -term contributions to scalar masses can appear as a dominant source of scalar mass splitting.

3.1. Sfermion mass formula

We consider the case with the intrinsic Z_2 parity assignment $(-1)^k \eta_k = (-1)^k \eta'_k = +1$. In the case with $(-1)^k \eta'_k = -1$, similar relations are derived by a suitable exchange of sfermion species. We list sfermion species as zero modes of five-dimensional fields with even Z_2 parities $\mathcal{P}_0 = \mathcal{P}_1 = +1$ in Table IV. In Table IV, \tilde{f} means the

Table IV. The assignment of sfermions and those $U(1)$ charges.

species	(l_1, l_2, l_3)	$l_1 + l_2$	$U(1)_2$	$U(1)_3$
\tilde{d}_R^*	(1, 0, even)	1	$-(N - 5)l_3 + r(k - 1)$	$-N + 5k$
\tilde{l}_L	(0, 1, odd)	1	$-(N - 5)l_3 + r(k - 1)$	$-N + 5k$
\tilde{u}_R^*	(2, 0, even)	2	$(N - 5)l_3 - r(k - 2)$	$2N - 5k$
\tilde{e}_R^*	(0, 2, even)	2	$(N - 5)l_3 - r(k - 2)$	$2N - 5k$
\tilde{q}_L	(1, 1, odd)	2	$(N - 5)l_3 - r(k - 2)$	$2N - 5k$
\tilde{e}_R^*	(3, 0, even)	3	$-(N - 5)l_3 + r(k - 3)$	$-3N + 5k$
\tilde{u}_R^*	(1, 2, even)	3	$-(N - 5)l_3 + r(k - 3)$	$-3N + 5k$
\tilde{q}_L	(2, 1, odd)	3	$-(N - 5)l_3 + r(k - 3)$	$-3N + 5k$
\tilde{l}_L	(3, 1, odd)	4	$(N - 5)l_3 - r(k - 4)$	$4N - 5k$
\tilde{d}_R^*	(2, 2, even)	4	$(N - 5)l_3 - r(k - 4)$	$4N - 5k$

scalar partner of fermion f , and the charge-conjugation is performed for the fields with $l_1 + l_2 = \text{odd}$. The asterisk stands for the complex conjugate. Note that the sign of $U(1)$ charges is changed by the charge-conjugation. As sfermion species are labeled by the numbers (l_1, l_2, l_3) , we use this label in place of \tilde{f} .

Sfermion mass squareds at M_U are written by

$$m_{(l_1, l_2, l_3)}^{(\alpha, \beta)}{}^2(M_U) = m_{[N, k]}^2 + (-1)^{l_1 + l_2} \sum_{A=1}^{r-1} Q_\alpha^A D_{(r)}^A + (-1)^{l_1 + l_2} \sum_{B=1}^{r-1} Q_\beta^B D_{(s)}^B \\ + (-1)^{l_1 + l_2} [(N - 5)l_3 - r(k - (l_1 + l_2))] D_2$$

^{*)} Scherk-Schwarz mechanism, in which SUSY is broken by the difference of BCs between bosons and fermions, is a typical one.²³⁾ This mechanism on S^1/Z_2 leads to a restricted type of soft SUSY breaking parameters such as $M_i = \beta/R$ for bulk gauginos and $m_{\tilde{f}}^2 = (\beta/R)^2$ for bulk scalar particles where β is a real parameter and R is a radius of S^1 .

$$+ (-1)^{l_1+l_2} [(l_1 + l_2)N - 5k] D_3, \quad (3.1)$$

where $m_{[N,k]}^2$ is a common soft SUSY breaking mass parameter which respects $SU(N)$ gauge symmetry and other terms in the right-hand side represent D -term contributions. The D -term contributions, in general, originate from D -terms related to broken gauge symmetries when soft SUSY breaking parameters possess a non-universal structure and the rank of gauge group lowers after the breakdown of gauge symmetry.^{9),24)} In most cases, the magnitude of D -term condensation is, at most, of order TeV scale squared and hence D -term contributions can induce sizable effects on sfermion spectrum. The α and β represent indices which indicate members of multiplet of $SU(r)$ and $SU(s)$, and run from 1 to ${}_r C_{l_3}$ and from 1 to ${}_s C_{l_4}$, respectively. The Q_α^A are broken diagonal charges of $[r, l_3]$, which form the Cartan sub-algebra of $SU(r)$, and given by

$$Q_\alpha^A = Q_\alpha^A([r, l_3]) = \sum_{a=a_1}^{a_{l_3}} Q_a^A, \quad (3.2)$$

where Q_a^A are the diagonal charges (up to normalization) for fields with the fundamental representation $[r, 1]$ defined by

$$Q_a^A \equiv (1 - a)\delta_{a-1}^A + \sum_{i=0}^{r-1-a} \delta_{a+i}^A. \quad (3.3)$$

The numbering for α is defined by

$$\begin{aligned} (a_1, \dots, a_{l_3}) &= (1, \dots, l_3) && \text{for } \alpha = 1 \\ &= (1, \dots, l_3 - 1, l_3 + 1) && \text{for } \alpha = 2 \\ &\dots && \\ &= (1, \dots, l_3 - 1, r) && \text{for } \alpha = l_3 - r + 1 \\ &= (1, \dots, l_3 - 2, l_3, l_3 + 1) && \text{for } \alpha = l_3 - r + 2 \\ &\dots && \\ &= (r + 1 - l_3, \dots, r) && \text{for } \alpha = {}_r C_{l_3}. \end{aligned} \quad (3.4)$$

By using formulae of diagonal charges (3.2) and (3.3) and the definition of numbering (3.4), the broken diagonal charges of $[r, r - l_3]$ (the complex conjugate representation of $[r, l_3]$) are given by

$$Q_\alpha^A([r, r - l_3]) = -Q_{{}_r C_{l_3} + 1 - \alpha}^A([r, l_3]). \quad (3.5)$$

The same holds on the charges Q_β^B . The $D_{(r)}^A$, $D_{(s)}^B$, D_2 and D_3 are parameters including D -term condensations and those magnitudes are model-dependent.

3.2. Sfermion mass relations

Let us derive relations among sfermion masses at M_U , by eliminating unknown parameters ($m_{[N,k]}^2$, $D_{(r)}^A$, $D_{(s)}^B$, D_2 , D_3) in the mass formula (3.1).

First of all, we find the following relations from the mass formula (3.1) and Table IV,

$$m_{(2,0,l_3)}^{(\alpha,\beta)^2} = m_{(0,2,l_3)}^{(\alpha,\beta)^2}, \quad m_{(3,0,l_3)}^{(\alpha,\beta)^2} = m_{(1,2,l_3)}^{(\alpha,\beta)^2}. \quad (3.6)$$

Here and hereafter we abbreviate $m_{(l_1,l_2,l_3)}^{(\alpha,\beta)^2}(M_U)$ as $m_{(l_1,l_2,l_3)}^{(\alpha,\beta)^2}$. This type of relation generally appears if up-type anti-squark singlet exists, and the number of relations is $n_{\bar{u}} (= n_{\bar{e}})$. Hereafter we consider only up-type anti-squark singlets (in place of positron-type slepton singlets).

Before we derive other relations, we estimate total number of independent relations. Number of each sfermion derived from bulk field $[N, k]$ equals to that of each fermion given in Eqs. (2.8) - (2.12). Total number of sfermions excluding slepton singlets is given by

$$N_{\text{tot}} = \sum_{i=1}^4 \sum_{l_3=0,1,\dots} r C_{l_3} \cdot N-5-r C_{k-i-l_3} = \sum_{i=1}^4 N-5 C_{k-i}. \quad (3.7)$$

The number of unknown parameters is $N - 4$ because the number of D -term condensations equals to the difference of rank between $SU(N)$ and G_{SM} . Hence the number of independent relations excluding (3.6) is $N_{\text{tot}} - N + 4$. We find that no relation is derived from $[N, 1]$ and one relation $m_{(2,0,0)}^{(\alpha,\beta)^2} = m_{(0,2,0)}^{(\alpha,\beta)^2}$ (the type (3.6)) from $[N, 2]$.

By taking the summation over all members in each multiplet of $SU(r)$ and $SU(s)$, the following formula is derived,

$$\begin{aligned} & \sum_{\alpha=1}^{r C_{l_3}} \sum_{\beta=1}^{s C_{l_4}} m_{(l_1,l_2,l_3)}^{(\alpha,\beta)^2} \\ &= r C_{l_3} \cdot s C_{l_4} \left(m_{[N,k]}^2 + (-1)^{l_1+l_2} [(N-5)l_3 - r(k - (l_1 + l_2))] D_2 \right. \\ & \quad \left. + (-1)^{l_1+l_2} [(l_1 + l_2)N - 5k] D_3 \right), \end{aligned} \quad (3.8)$$

Note that both $D_{(r)}^A$ and $D_{(s)}^B$ disappear because of the traceless property of diagonal generators. If the number of multiplets (N_{mul}) is beyond three, $N_{\text{mul}} - 3$ kinds of relations are derived by eliminating unknown parameters ($m_{[N,k]}^2$, D_2 , D_3).

The remaining relations are derived by a summation among multiplets with suitable coefficients (not a universal one), and are formally written by

$$\sum_{\alpha} c_{\alpha} m_{(l_1,l_2,l_3)}^{(\alpha,\beta)^2} = \sum_{\alpha'} c'_{\alpha'} m_{(l'_1,l'_2,l'_3)}^{(\alpha',\beta')^2}, \quad \sum_{\beta} d_{\beta} m_{(l_1,l_2,l_3)}^{(\alpha,\beta)^2} = \sum_{\beta'} d'_{\beta'} m_{(l'_1,l'_2,l'_3)}^{(\alpha',\beta')^2}, \quad (3.9)$$

where c_{α} , $c'_{\alpha'}$, d_{β} and $d'_{\beta'}$ are coefficients which satisfies the following relations,

$$\sum_{\alpha} c_{\alpha} = \sum_{\alpha'} c'_{\alpha'}, \quad \sum_{\alpha} c_{\alpha} Q_{\alpha}^A = \sum_{\alpha'} c'_{\alpha'} Q_{\alpha'}^A, \quad (3.10)$$

$$\sum_{\beta} d_{\beta} = \sum_{\beta'} d'_{\beta'}, \quad \sum_{\beta} d_{\beta} Q_{\beta}^A = \sum_{\beta'} d'_{\beta'} Q_{\beta'}^A. \quad (3.11)$$

Sfermion mass relations (excluding the type (3·6)) derived from [6, 3] - [9, 4] are listed in Table V. We have classified mass relations into three types, but the form

Table V. The sfermion mass relations derived from [6, 3] - [9, 4].

rep.	(p, q, r, s)	sfermion mass relations
[6, 3]	$(3, 2, 1, 0)$	$m_{(1,1,1)}^{(1,1)2} = m_{(1,2,0)}^{(1,1)2}$
	$(3, 2, 0, 1)$	$m_{(2,0,0)}^{(1,1)2} = m_{(1,2,0)}^{(1,1)2}$
[7, 3]	$(3, 2, 2, 0)$	$5m_{(1,0,2)}^{(1,1)2} + 9m_{(1,2,0)}^{(1,1)2} = 7 \sum_{\alpha=1}^2 m_{(1,1,1)}^{(\alpha,1)2}$
	$(3, 2, 1, 1)$	$5m_{(0,1,1)}^{(1,1)2} + 9m_{(1,2,0)}^{(1,1)2} = 7 \left(m_{(1,1,1)}^{(1,1)2} + m_{(2,0,0)}^{(1,1)2} \right)$
	$(3, 2, 0, 2)$	$5m_{(1,0,0)}^{(1,1)2} + 9m_{(1,2,0)}^{(1,1)2} = 7 \sum_{\beta=1}^2 m_{(2,0,0)}^{(1,\beta)2}$
[8, 3]	$(3, 2, 3, 0)$	$5 \sum_{\alpha=1}^3 m_{(1,0,2)}^{(\alpha,1)2} + 9m_{(1,2,0)}^{(1,1)2} = 8 \sum_{\alpha=1}^3 m_{(1,1,1)}^{(\alpha,1)2},$ $m_{(1,0,2)}^{(3,1)2} - m_{(1,1,1)}^{(1,1)2} = m_{(1,0,2)}^{(2,1)2} - m_{(1,1,1)}^{(2,1)2} = m_{(1,0,2)}^{(1,1)2} - m_{(1,1,1)}^{(3,1)2}$
	$(3, 2, 2, 1)$	$\sum_{\alpha=1}^2 m_{(1,1,1)}^{(\alpha,1)2} + 2m_{(1,0,2)}^{(1,1)2} = \sum_{\alpha=1}^2 m_{(0,1,1)}^{(\alpha,1)2} + 2m_{(2,0,0)}^{(1,1)2},$ $6m_{(2,0,0)}^{(1,1)2} + \sum_{\alpha=1}^2 m_{(1,1,1)}^{(\alpha,1)2} = 5m_{(1,0,2)}^{(1,1)2} + 3m_{(1,2,0)}^{(1,1)2},$ $m_{(0,1,1)}^{(1,1)2} - m_{(0,1,1)}^{(2,1)2} = m_{(1,1,1)}^{(2,1)2} - m_{(1,1,1)}^{(1,1)2}$
	$(3, 2, 1, 2)$	$\sum_{\beta=1}^2 m_{(2,0,0)}^{(1,\beta)2} + 2m_{(1,0,0)}^{(1,1)2} = \sum_{\beta=1}^2 m_{(0,1,1)}^{(1,\beta)2} + 2m_{(1,1,1)}^{(1,1)2},$ $6m_{(1,1,1)}^{(1,1)2} + \sum_{\beta=1}^2 m_{(2,0,0)}^{(1,\beta)2} = 5m_{(1,0,0)}^{(1,1)2} + 3m_{(1,2,0)}^{(1,1)2},$ $m_{(0,1,1)}^{(1,1)2} - m_{(0,1,1)}^{(1,2)2} = m_{(2,0,0)}^{(1,2)2} - m_{(2,0,0)}^{(1,1)2}$
	$(3, 2, 0, 3)$	$5 \sum_{\beta=1}^3 m_{(1,0,0)}^{(1,\beta)2} + 9m_{(1,2,0)}^{(1,1)2} = 8 \sum_{\beta=1}^3 m_{(2,0,0)}^{(1,\beta)2},$ $m_{(1,0,0)}^{(1,3)2} - m_{(2,0,0)}^{(1,1)2} = m_{(1,0,0)}^{(1,2)2} - m_{(2,0,0)}^{(1,2)2} = m_{(1,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,3)2}$
	$(3, 2, 3, 0)$	$m_{(0,1,3)}^{(1,1)2} = m_{(2,2,0)}^{(1,1)2},$ $m_{(2,0,2)}^{(1,1)2} = m_{(2,1,1)}^{(3,1)2}, m_{(2,0,2)}^{(2,1)2} = m_{(2,1,1)}^{(2,1)2}, m_{(2,0,2)}^{(3,1)2} = m_{(2,1,1)}^{(1,1)2}$
[8, 4]	$(3, 2, 2, 1)$	$m_{(1,0,2)}^{(1,1)2} = m_{(2,2,0)}^{(1,1)2}, m_{(2,0,2)}^{(1,1)2} = m_{(1,2,0)}^{(1,1)2},$ $m_{(1,1,1)}^{(1,1)2} = m_{(2,1,1)}^{(2,1)2}, m_{(1,1,1)}^{(2,1)2} = m_{(2,1,1)}^{(1,1)2}$
	$(3, 2, 1, 2)$	$m_{(0,1,1)}^{(1,1)2} = m_{(2,2,0)}^{(1,1)2}, m_{(2,0,0)}^{(1,1)2} = m_{(2,1,1)}^{(1,1)2},$ $m_{(1,1,1)}^{(1,1)2} = m_{(1,2,0)}^{(1,2)2}, m_{(1,1,1)}^{(1,2)2} = m_{(1,2,0)}^{(1,1)2}$
	$(3, 2, 0, 3)$	$m_{(1,0,0)}^{(1,1)2} = m_{(2,2,0)}^{(1,1)2},$ $m_{(2,0,0)}^{(1,1)2} = m_{(1,2,0)}^{(1,3)2}, m_{(2,0,0)}^{(1,3)2} = m_{(1,2,0)}^{(1,1)2}$
	$(3, 2, 3, 0)$	$m_{(1,0,0)}^{(1,1)2} = m_{(2,2,0)}^{(1,1)2},$ $m_{(2,0,0)}^{(1,1)2} = m_{(1,2,0)}^{(1,3)2}, m_{(2,0,0)}^{(1,3)2} = m_{(1,2,0)}^{(1,1)2}$

of mass relations is not unique. For example, we derive the second type relation such as $\sum_{\alpha=1}^3 m_{(2,0,2)}^{(\alpha,1)2} = \sum_{\alpha=1}^3 m_{(2,1,1)}^{(\alpha,1)2}$ and two third type relations $m_{(2,0,2)}^{(1,1)2} = m_{(2,1,1)}^{(3,1)2}$ and $m_{(2,0,2)}^{(2,1)2} = m_{(2,1,1)}^{(2,1)2}$ from [8, 4] for (3, 2, 3, 0). By using them, three third type rela-

rep.	(p, q, r, s)	sfermion mass relations
[9, 3]	$(3, 2, 4, 0)$	$9 \sum_{\alpha=1}^4 m_{(1,1,1)}^{(\alpha,1)2} = 5 \sum_{\alpha=1}^6 m_{(1,0,2)}^{(\alpha,1)2} + 6m_{(1,2,0)}^{(1,1)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(2,1)2} = m_{(1,0,2)}^{(4,1)2} - m_{(1,0,2)}^{(2,1)2} = m_{(1,0,2)}^{(5,1)2} - m_{(1,0,2)}^{(3,1)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(3,1)2} = m_{(1,0,2)}^{(4,1)2} - m_{(1,0,2)}^{(1,1)2} = m_{(1,0,2)}^{(6,1)2} - m_{(1,0,2)}^{(3,1)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(4,1)2} = m_{(1,0,2)}^{(5,1)2} - m_{(1,0,2)}^{(1,1)2}$
	$(3, 2, 3, 1)$	$\sum_{\alpha=1}^3 m_{(1,0,2)}^{(\alpha,1)2} + \sum_{\alpha=1}^3 m_{(1,1,1)}^{(\alpha,1)2} = \sum_{\alpha=1}^3 m_{(0,1,1)}^{(\alpha,1)2} + 3m_{(2,0,0)}^{(1,1)2},$ $\sum_{\alpha=1}^3 m_{(1,0,2)}^{(\alpha,1)2} + 4 \sum_{\alpha=1}^3 m_{(0,1,1)}^{(\alpha,1)2} + 3m_{(1,2,0)}^{(1,1)2} = 6 \sum_{\alpha=1}^3 m_{(1,1,1)}^{(\alpha,1)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(2,1)2} = m_{(1,0,2)}^{(3,1)2} - m_{(1,0,2)}^{(2,1)2} = m_{(0,1,1)}^{(2,1)2} - m_{(0,1,1)}^{(1,1)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(3,1)2} = m_{(1,0,2)}^{(3,1)2} - m_{(1,0,2)}^{(1,1)2} = m_{(0,1,1)}^{(3,1)2} - m_{(0,1,1)}^{(1,1)2}$
	$(3, 2, 2, 2)$	$m_{(1,0,2)}^{(1,1)2} + \sum_{\alpha=1}^2 m_{(1,1,1)}^{(\alpha,1)2} = m_{(1,0,0)}^{(1,1)2} + \sum_{\beta=1}^2 m_{(2,0,0)}^{(1,\beta)2},$ $3 \left(\sum_{\alpha=1}^2 m_{(1,1,1)}^{(\alpha,1)2} + \sum_{\beta=1}^2 m_{(2,0,0)}^{(1,\beta)2} \right) = 2m_{(1,2,0)}^{(1,1)2} + 5 \left(m_{(1,0,0)}^{(1,1)2} + m_{(1,0,2)}^{(1,1)2} \right),$ $2m_{(1,0,0)}^{(1,1)2} + 2m_{(1,0,2)}^{(1,1)2} = \sum_{\alpha=1}^2 \sum_{\beta=1}^2 m_{(0,1,1)}^{(\alpha,\beta)2},$ $m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(2,1)2} = m_{(0,1,1)}^{(2,1)2} - m_{(0,1,1)}^{(1,1)2} = m_{(0,1,1)}^{(2,2)2} - m_{(0,1,1)}^{(1,2)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,2)2} = m_{(0,1,1)}^{(1,2)2} - m_{(0,1,1)}^{(1,1)2}$
	$(3, 2, 1, 3)$	$\sum_{\beta=1}^3 m_{(1,0,0)}^{(1,\beta)2} + \sum_{\beta=1}^3 m_{(2,0,0)}^{(1,\beta)2} = \sum_{\beta=1}^3 m_{(0,1,1)}^{(1,\beta)2} + 3m_{(1,1,1)}^{(1,1)2},$ $\sum_{\beta=1}^3 m_{(1,0,0)}^{(1,\beta)2} + 4 \sum_{\beta=1}^3 m_{(0,1,1)}^{(1,\beta)2} + 3m_{(1,2,0)}^{(1,1)2} = 6 \sum_{\beta=1}^3 m_{(2,0,0)}^{(1,\beta)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,2)2} = m_{(1,0,0)}^{(1,3)2} - m_{(1,0,0)}^{(1,2)2} = m_{(0,1,1)}^{(1,2)2} - m_{(0,1,1)}^{(1,1)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,3)2} = m_{(1,0,0)}^{(1,3)2} - m_{(1,0,0)}^{(1,1)2} = m_{(0,1,1)}^{(1,3)2} - m_{(0,1,1)}^{(1,1)2}$
	$(3, 2, 0, 4)$	$9 \sum_{\beta=1}^4 m_{(2,0,0)}^{(1,\beta)2} = 5 \sum_{\beta=1}^6 m_{(1,0,0)}^{(1,\beta)2} + 6m_{(1,2,0)}^{(1,1)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,2)2} = m_{(1,0,0)}^{(1,4)2} - m_{(1,0,0)}^{(1,2)2} = m_{(1,0,0)}^{(1,5)2} - m_{(1,0,0)}^{(1,3)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,3)2} = m_{(1,0,0)}^{(1,4)2} - m_{(1,0,0)}^{(1,1)2} = m_{(1,0,0)}^{(1,6)2} - m_{(1,0,0)}^{(1,3)2},$ $m_{(2,0,0)}^{(1,1)2} - m_{(2,0,0)}^{(1,4)2} = m_{(1,0,0)}^{(1,5)2} - m_{(1,0,0)}^{(1,1)2}$

tions are written down in Table V. Mass relations derived from [9, 4] for $(p, q, r, s) = (3, 2, 1, 3)$ are obtained from those for $(p, q, r, s) = (3, 2, 3, 1)$ by the following replacement,

$$\begin{aligned}
m_{(1,0,2)}^{(\alpha,1)2} &\rightarrow m_{(1,2,0)}^{(1,\beta)2}, \quad m_{(0,1,3)}^{(1,1)2} \rightarrow m_{(2,1,1)}^{(1,1)2}, \quad m_{(2,0,2)}^{(\alpha,1)2} \rightarrow m_{(1,1,1)}^{(1,\beta)2}, \\
m_{(1,1,1)}^{(\alpha,1)2} &\rightarrow m_{(2,0,0)}^{(1,\beta)2}, \quad m_{(1,2,0)}^{(1,1)2} \rightarrow m_{(1,0,0)}^{(1,1)2}, \quad m_{(2,1,1)}^{(\alpha,1)2} \rightarrow m_{(0,1,1)}^{(1,\beta)2}, \\
m_{(2,2,0)}^{(1,1)2} &\rightarrow m_{(2,2,0)}^{(1,1)2}.
\end{aligned} \tag{3.12}$$

rep.	(p, q, r, s)	sfermion mass relations
[9, 4]	$(3, 2, 4, 0)$	$12 \sum_{\alpha=1}^6 m_{(2,0,2)}^{(\alpha,1)2} = 5 \sum_{\alpha=1}^4 m_{(0,1,3)}^{(\alpha,1)2} + 13 \sum_{\alpha=1}^4 m_{(2,1,1)}^{(\alpha,1)2},$ $23 \sum_{\alpha=1}^6 m_{(2,0,2)}^{(\alpha,1)2} = 27 \sum_{\alpha=1}^4 m_{(2,1,1)}^{(\alpha,1)2} + 30 m_{(2,2,0)}^{(1,1)2},$ $m_{(0,1,3)}^{(2,1)2} - m_{(0,1,3)}^{(1,1)2} = m_{(2,1,1)}^{(4,1)2} - m_{(2,1,1)}^{(3,1)2}$ $= m_{(2,0,2)}^{(2,1)2} - m_{(2,0,2)}^{(4,1)2} = m_{(2,0,2)}^{(3,1)2} - m_{(2,0,2)}^{(5,1)2},$ $m_{(0,1,3)}^{(3,1)2} - m_{(0,1,3)}^{(1,1)2} = m_{(2,1,1)}^{(4,1)2} - m_{(2,1,1)}^{(2,1)2}$ $= m_{(2,0,2)}^{(1,1)2} - m_{(2,0,2)}^{(4,1)2} = m_{(2,0,2)}^{(3,1)2} - m_{(2,0,2)}^{(6,1)2},$ $m_{(0,1,3)}^{(4,1)2} - m_{(0,1,3)}^{(1,1)2} = m_{(2,1,1)}^{(4,1)2} - m_{(2,1,1)}^{(1,1)2}$ $= m_{(2,0,2)}^{(1,1)2} - m_{(2,0,2)}^{(5,1)2}$
	$(3, 2, 3, 1)$	$\sum_{\alpha=1}^3 m_{(1,0,2)}^{(\alpha,1)2} - 3m_{(0,1,3)}^{(1,1)2} = \sum_{\alpha=1}^3 m_{(2,0,2)}^{(\alpha,1)2} - \sum_{\alpha=1}^3 m_{(1,1,1)}^{(\alpha,1)2}$ $= 3m_{(1,2,0)}^{(1,1)2} - \sum_{\alpha=1}^3 m_{(2,1,1)}^{(\alpha,1)2},$ $27 \left(m_{(1,2,0)}^{(1,1)2} + m_{(0,1,3)}^{(1,1)2} \right) = 12m_{(2,2,0)}^{(1,1)2} + 7 \left(\sum_{\alpha=1}^3 m_{(2,0,2)}^{(\alpha,1)2} + \sum_{\alpha=1}^3 m_{(1,1,1)}^{(\alpha,1)2} \right),$ $8 \sum_{\alpha=1}^3 m_{(1,0,2)}^{(\alpha,1)2} + \sum_{\alpha=1}^3 m_{(2,1,1)}^{(\alpha,1)2},$ $= m_{(0,1,3)}^{(1,1)2} + 8m_{(1,2,0)}^{(1,1)2} + 18m_{(2,2,0)}^{(1,1)2}$ $m_{(1,0,2)}^{(1,1)2} - m_{(1,0,2)}^{(2,1)2} = m_{(1,1,1)}^{(3,1)2} - m_{(1,1,1)}^{(2,1)2}$ $= m_{(2,1,1)}^{(2,1)2} - m_{(2,1,1)}^{(3,1)2} = m_{(2,0,2)}^{(2,1)2} - m_{(2,0,2)}^{(1,1)2},$ $m_{(1,0,2)}^{(1,1)2} - m_{(1,0,2)}^{(3,1)2} = m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(3,1)2}$ $= m_{(2,1,1)}^{(3,1)2} - m_{(2,1,1)}^{(1,1)2} = m_{(2,0,2)}^{(3,1)2} - m_{(2,0,2)}^{(1,1)2}$
	$(3, 2, 2, 2)$	$\sum_{\alpha=1}^2 m_{(2,1,1)}^{(\alpha,1)2} - \sum_{\beta=1}^2 m_{(1,2,0)}^{(1,\beta)2} = \sum_{\beta=1}^2 m_{(1,0,2)}^{(1,\beta)2} - \sum_{\alpha=1}^2 m_{(0,1,1)}^{(\alpha,1)2}$ $= m_{(2,0,0)}^{(1,1)2} - m_{(2,0,2)}^{(1,1)2},$ $\sum_{\alpha=1}^2 m_{(2,1,1)}^{(\alpha,1)2} + \sum_{\alpha=1}^2 m_{(0,1,1)}^{(\alpha,1)2} = \sum_{\beta=1}^2 m_{(1,0,2)}^{(1,\beta)2} + \sum_{\beta=1}^2 m_{(1,2,0)}^{(1,\beta)2},$ $\sum_{\alpha=1}^2 \sum_{\beta=1}^2 m_{(1,1,1)}^{(\alpha,\beta)2} = 2m_{(2,0,2)}^{(1,1)2} + 2m_{(2,0,0)}^{(1,1)2},$ $m_{(1,2,0)}^{(1,1)2} - m_{(1,2,0)}^{(1,2)2} = m_{(1,0,2)}^{(1,1)2} - m_{(1,0,2)}^{(1,2)2}$ $= m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(1,2)2} = m_{(1,1,1)}^{(2,1)2} - m_{(1,1,1)}^{(2,2)2},$ $m_{(2,1,1)}^{(1,1)2} - m_{(2,1,1)}^{(2,1)2} = m_{(0,1,1)}^{(1,1)2} - m_{(0,1,1)}^{(2,1)2}$ $= m_{(1,1,1)}^{(1,1)2} - m_{(1,1,1)}^{(2,1)2} = m_{(1,1,1)}^{(1,2)2} - m_{(1,1,1)}^{(2,2)2}$

In the same way, mass relations derived from [9, 4] for $(p, q, r, s) = (3, 2, 0, 4)$ are obtained from those for $(p, q, r, s) = (3, 2, 4, 0)$ by the following replacement,

$$\begin{aligned}
m_{(2,0,2)}^{(\alpha,1)2} &\rightarrow m_{(2,0,0)}^{(1,\beta)2}, \quad m_{(0,1,3)}^{(\alpha,1)2} \rightarrow m_{(1,0,0)}^{(1,\beta)2}, \\
m_{(2,1,1)}^{(\alpha,1)2} &\rightarrow m_{(1,2,0)}^{(1,\beta)2}, \quad m_{(2,2,0)}^{(1,1)2} \rightarrow m_{(2,2,0)}^{(1,1)2}.
\end{aligned} \tag{3.13}$$

We have obtained mass relations among sfermions which stem from the bulk field with $[N, k]$. Those are specific to each $[N, k]$ and gauge symmetry breaking pattern, and can be useful probes to select models.

Brane fields at $y = 0$ are $SU(5) \times SU(N - 5)$ multiplets, and those soft masses satisfy the $SU(5)$ GUT relations,

$$m_{\tilde{q}_L}^2 = m_{\tilde{u}_R^*}^2 = m_{\tilde{e}_R^*}^2, \quad m_{\tilde{l}_L}^2 = m_{\tilde{d}_R^*}^2. \quad (3.14)$$

So far we assume that all zero modes survive after the breakdown of extra gauge symmetries. In case that particle mixing and/or decoupling occurs, some relations should be modified. We need further model-dependent analyses to derive specific relations in such a case.

§4. Conclusions

We have studied sfermion masses from a general framework, based on orbifold family unification models under some assumptions regarding the breakdown of SUSY and gauge symmetries, and derived relations among them. Sfermion mass relations are specific to each model and can be useful for a selection of realistic model.

A non-abelian subgroup such as $SU(r) \times SU(s)$ of $SU(N)$ plays the role of family symmetry and its D -term contributions spoil the mass degeneracy. Conversely, the requirement of degenerate masses would give a constraint on the D -term condensations and/or SUSY breaking mechanism. For example, if we take Scherk-Schwarz mechanism for $N = 1$ SUSY breaking, the D -term condensations vanish for the gauge symmetries broken at the orbifold breaking scale M_U because of a universal structure of soft SUSY breaking parameters. If extra gauge symmetries, however, are broken at different scales from M_U , soft SUSY breaking parameters receive extra renormalization effects and turn out a non-universal structure. As a result, D -term contributions can appear. In this case, our analysis should be modified by considering the renormalization group running for sfermion masses. In the case that effects such as F -term contributions and/or higher dimensional operators are sizable, we should consider them.

Sum rules of sparticle masses at the TeV scale can be derived if the physics between the breaking scale M_U and the weak scale is specified. In our previous analysis, we have assumed the gravity-mediated SUSY breaking in the case that the dynamics in the hidden sector do not give sizable effects on renormalization group evolutions of soft SUSY breaking parameters.¹⁸⁾ It is also important to study the case with strong dynamics in the hidden sector.²⁶⁾

Acknowledgements

This work was supported in part by Scientific Grants from the Ministry of Education and Science, Grant No.18204024, Grant No.18540259 (Y.K.).

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